POLS 7012: Problem Set 6 (Calculus)

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Problem 1

A dictator is deciding how much tax revenue to steal for himself and how much to give to his generals. He can steal some fraction of revenue x between 0 and 1. If he steals all of it ($x = 1$), then his generals will get mad, overthrow him, and he gets nothing. If he gives everything to his generals ($x = 0$), then his generals are happy, don't overthrow him, but he still gets nothing because he gave away all the loot.

If the probability that the generals leave him in place equals $1 - x$, how much should the dictator steal to maximize his **expected happiness** (probability of staying in power $\times$ the amount of money stolen)?

Problem 2

Suppose you have three data points: x_1, x_2 and x_3. Find the value m that minimizes the sum of squared errors $f(m)$:

$$f(m) = (x_1 - m)^2 + (x_2 - m)^2 + (x_3 - m)^2$$

Problem 3

Consider the following regression model:

$$Y = 2X_1 + 5X_2 - 4X_1X_2 + \varepsilon$$

If you increase X_1 by 1 unit, what happens to Y?